“A change by a cause”:

advancing precision cosmology from Ptolemaic to Copernican

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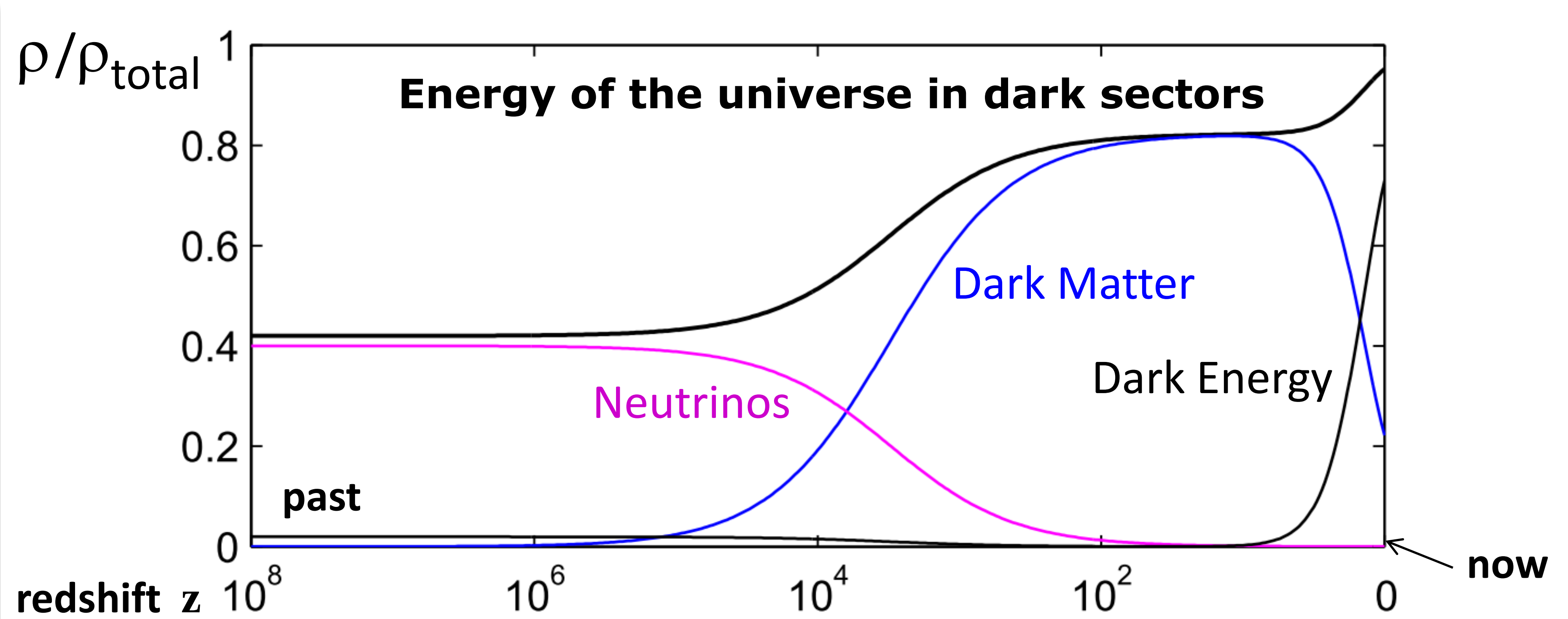
2007, revised in 2021

Motivation



The properties of the dark sectors are probed best by observing signatures of dark species' **inhomogeneities**.

Detectable signatures of dynamics of dark inhomogeneities:

- ✓ break degeneracy of constraints from background evolution,
- ✓ provide new information at every scale k.

Crucial subtleties

Which properties of the dark species are constrained by observations?
Which data probes which properties of the dark species?

At any scale k, we should address **horizon entry**, when $\mathcal{H}(z) \equiv \frac{a_{,\tau}}{a} \sim k$

- ✓ Only during the entry can perturbations in dark species with $p \sim \rho$ (e.g., neutrinos, dynamical dark energy, etc.) reveal themselves.*
- ✓ During the entry, perturbations in any dark species affect the CMB much stronger than at any other time.

Yet before and during the horizon entry

description of evolution of perturbations is highly ambiguous

(“Gauge-invariant” formalisms neither eliminate nor alleviate the ambiguity – see slide 6.)

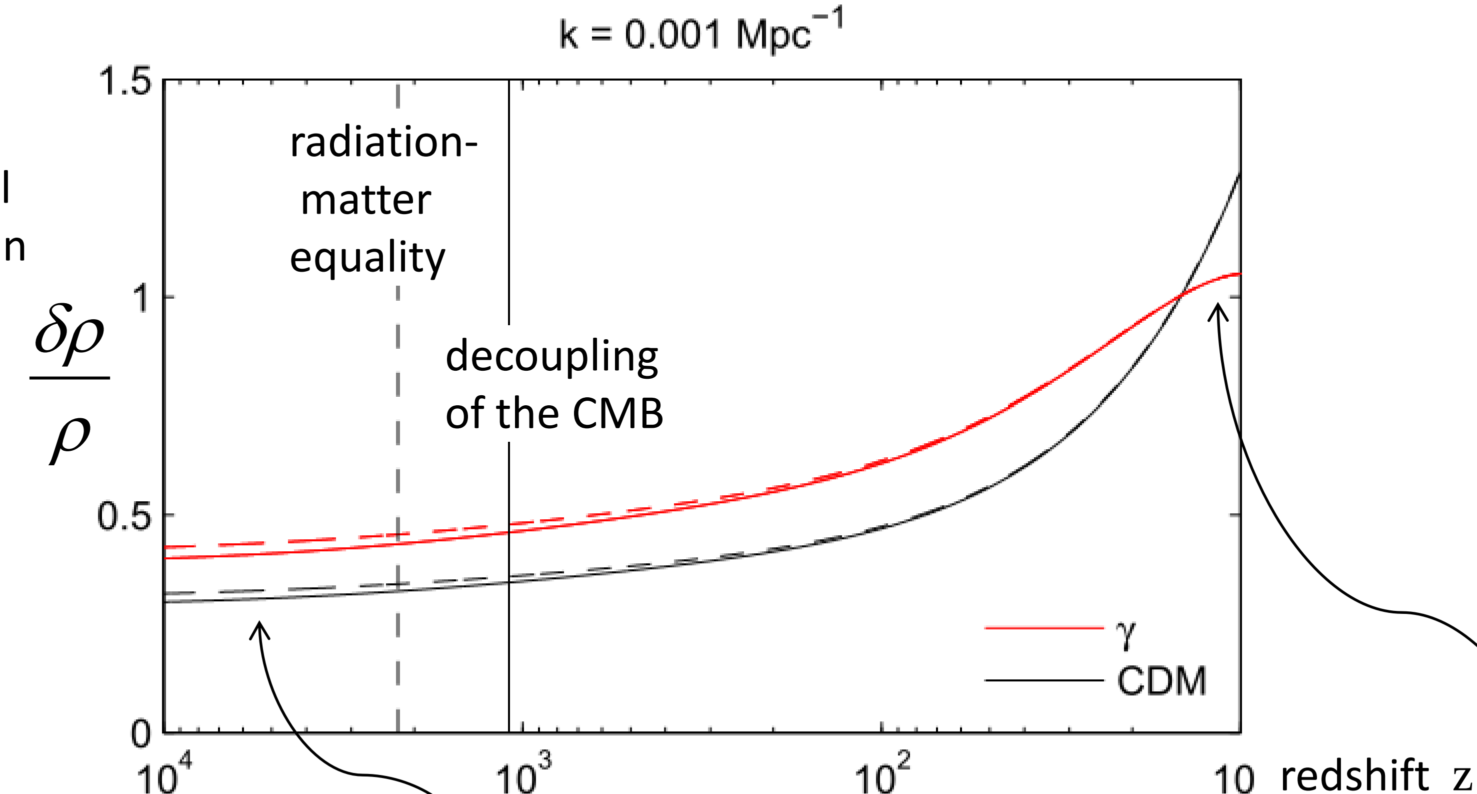
* Indeed, after the entry, when the spatial scale of a perturbation $l = k^{-1}$ has become smaller than the Hubble scale $l_H = \mathcal{H}^{-1}$, any such species, whose overdensity remains small, contribute to the gravitational potential negligibly: $\delta\Phi \sim -\left(\frac{l}{l_H}\right)^2 \frac{\delta\rho}{\rho}$

Example of descriptional ambiguities

Compare two models of extra dark radiation, additional to neutrinos: new decoupled particles (solid) vs. tracking quintessence (dashed).

The plots show for the two modes, with same inflationary init. conditions, evolution of perturbations that **enter** the horizon **in the matter era:**

(all in conformal Newtonian gauge)

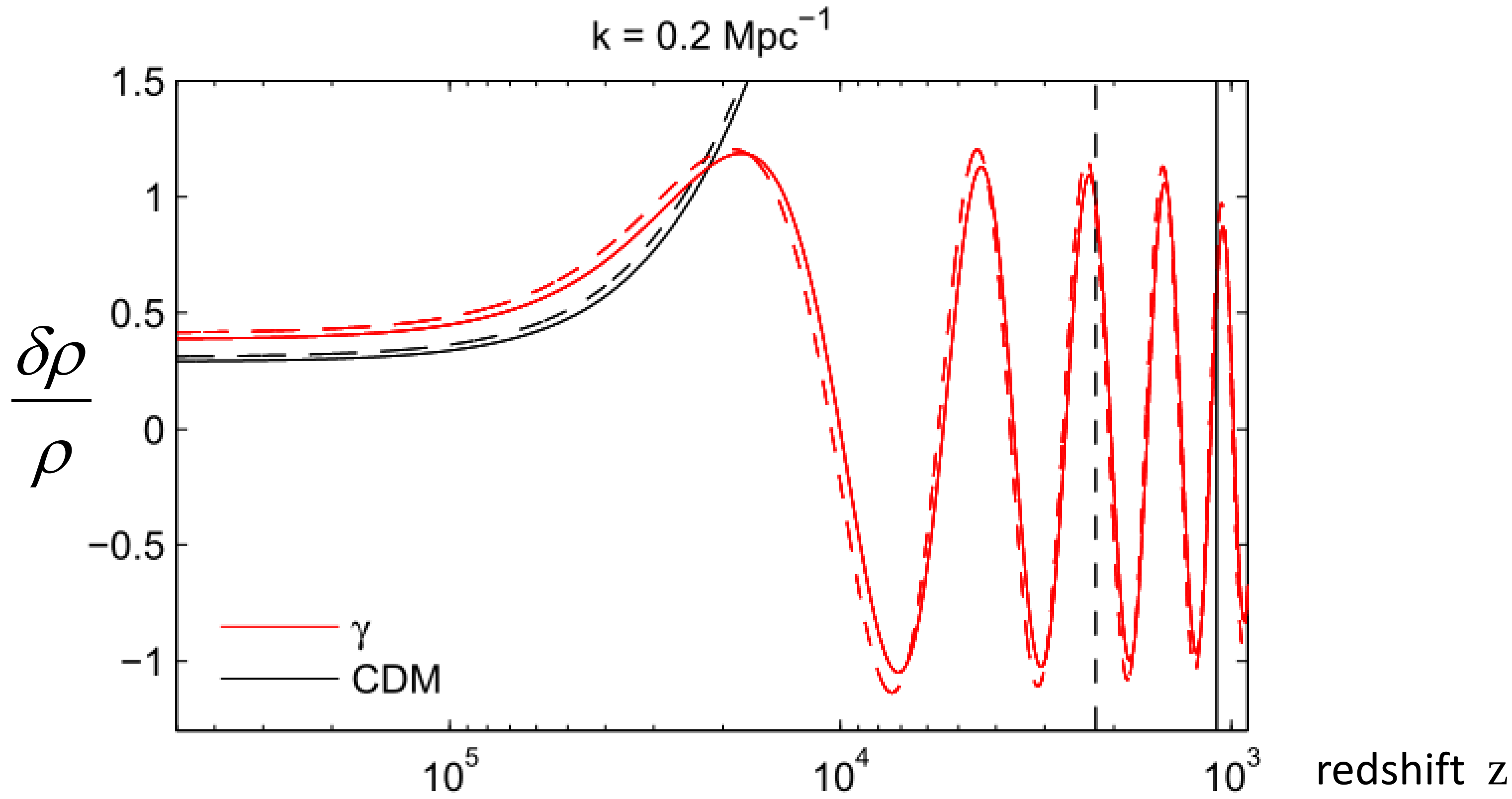


The apparent difference of the evolution in the compared models is a descriptional artifact.

Indeed, the two models yield the **same observables**.

Example of descriptional ambiguities, continued

Yet for the same two models with the same inflationary initial conditions, compare perturbations that **enter in the radiation era**:



At this scale k, **observables differ among the two models!**

How to separate physics from artifacts of its description?

Alternative approaches

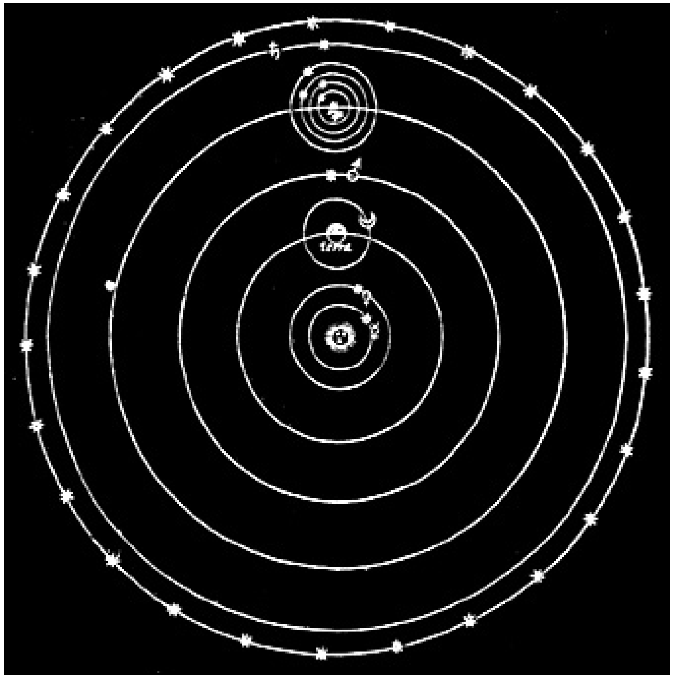
A. “Gauge-invariant” descriptions:
   – are equivalent to gauge fixing, thus are **equally ambiguous**.

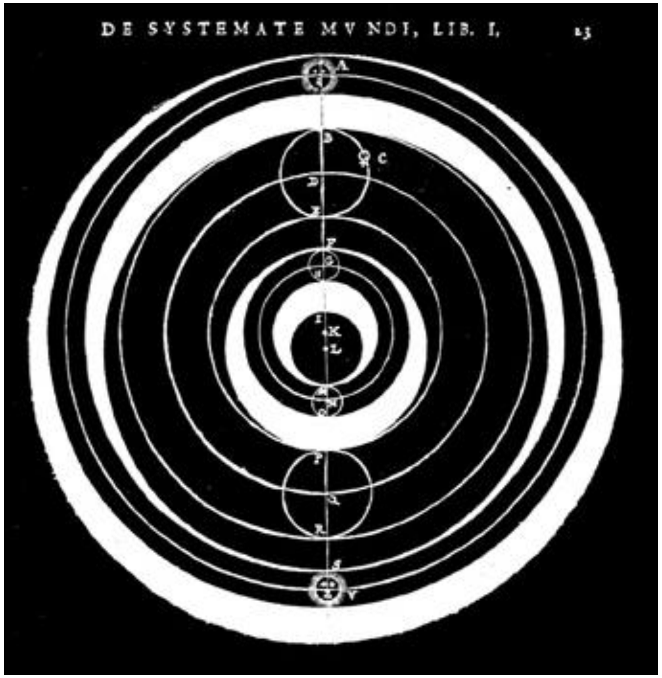
B. Numerical computations of observables in any fixed gauge:
   – important yet visually inconspicuous signatures of dark species and of their properties are **easy to overlook**;
   – the origin of various features of the observable distributions is established by **guessing**
     (often **wrong**).

C. Dynamical variables that reveal causal dependences explicitly.

Example: Weakly perturbed Minkowski metric

When the metric is almost Minkowski then in certain, *preferred coordinates* – those of the inertial frames – any **changes** of velocities are necessarily **due to** objective external **causes**:





The Copernican, as opposed to Ptolemaic, variables offer:

- ✓ Simpler description of dynamics and evolution
- ✓ Manifest "cause ↔ effect" relations
- ✓ Lead to underlying, more fundamental laws

Large-scale linear dynamics in FRW metric

When the metric is almost FRW, i.e., is weakly perturbed FRW,

1. Do objective "cause $\leftrightarrow$ effect" dependencies exist?
2. If yes, do any variables show the objective dependencies manifestly?
3. Does this lead to physics more fundamental than we currently know?

"Yes" for 1. and 2. when perturbations evolve linearly:

- ✓ Subhorizon evolution: Use variables that become the perturbations in inertial frames (with almost Minkowski metric).
- ✓ Superhorizon: If a perturbation couples only gravitationally then its observable impact does not depend on events that happened when the perturbation was superhorizon. Hence, use variables that are frozen while the perturbation is superhorizon.
- ✓ Horizon entry: Next slide

Weakly perturbed Minkowski space $\leftrightarrow$ Linearly perturbed FRW

$\exists$ coordinate frames in which:

The velocity an object is **constant** when no force from an objectively identifiable external source acts on the object

$\exists$ measures of overdensities of cosmological species that are:

1. **Frozen** on superhorizon scales
   (unless the species are being created by other species with different overdensity)
2. Remain constant (i.e., **unaffected by spurious "gravitational forces"**) in homogeneous and isotropic geometry
3. Reduce to the ordinary proper excess of density on subhorizon scales

We can prove

a. Uniqueness: All measures of species' overdensity that satisfy 1–3 and coincide on small scales must also coincide on superhorizon scales.

b. Cause-effect faithfulness: A change in an evolving variable that satisfies 1–3 is **concurrent** with the microscopic processes responsible for the change.
(Fails for the majority of the traditional variables!)

SB, Phys. Rev. D 74 (2006)

Measures that satisfy 1-3

- **Canonical** phase-space distribution:

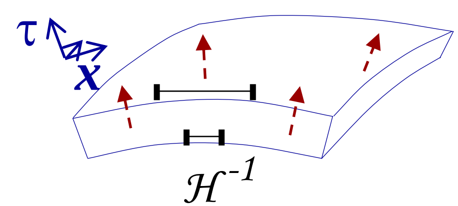
$$\delta f(x^i, P_i), \qquad P_i \text{ are the canonical (not the physical) momenta}$$

- **Canonical** radiation intensity:

$$\iota(x^i, n_i) \equiv \frac{I(x^i, n_i) - \bar{I}}{\bar{I}}, \qquad I(x^i, n_i) \equiv \int_0^\infty P^3 dP\, f(x^i, n_i P)$$

- **Coordinate** density of a “conserved particle number”:

$$\delta_a \equiv \frac{\delta n_a}{n_a} = \frac{\delta\rho_a}{\rho_a + p_a} - 3\Psi \quad \left(= 3\zeta_a\right)$$



(for any gauge without shear, with $\delta g_{ij}^{(3)} = -2a^2\delta_{ij}\Psi$)

✓ These quantities are interrelated:

$$\delta(x^i) = \frac{\int d^3P_i\, P_0\, \delta f(x^i, P_i)}{a^4(\rho + p)} = \frac{3}{4}\left\langle \iota(x^i, n_i)\right\rangle_{n_i}$$

This approach	vs.	Traditional

- Photon fluid: $\ddot{\delta}_\gamma + \chi\dot{\delta}_\gamma - c_s^2\nabla^2\delta_\gamma =$

$$= \nabla^2(\Phi + \Psi)$$

$$= \nabla^2\Phi + 3\ddot{\Psi}$$

(Easy to include scattering and polarization: SB, PRD 2006)

- Massive matter: $\ddot{\delta}_c + \mathcal{H}\dot{\delta}_c =$

$$= \nabla^2\Phi$$

$$= \nabla^2\Phi + 3\mathcal{H}\dot{\Psi} + 3\ddot{\Psi}$$

(Easy to include scattering: SB, PRD 2006)

- Intensity of streaming (decoupled) relativistic neutrinos or photons:

$$\dot{\imath} + n_i\nabla_i\,\imath =$$

$$-4n_i\nabla_i(\Phi + \Psi)$$

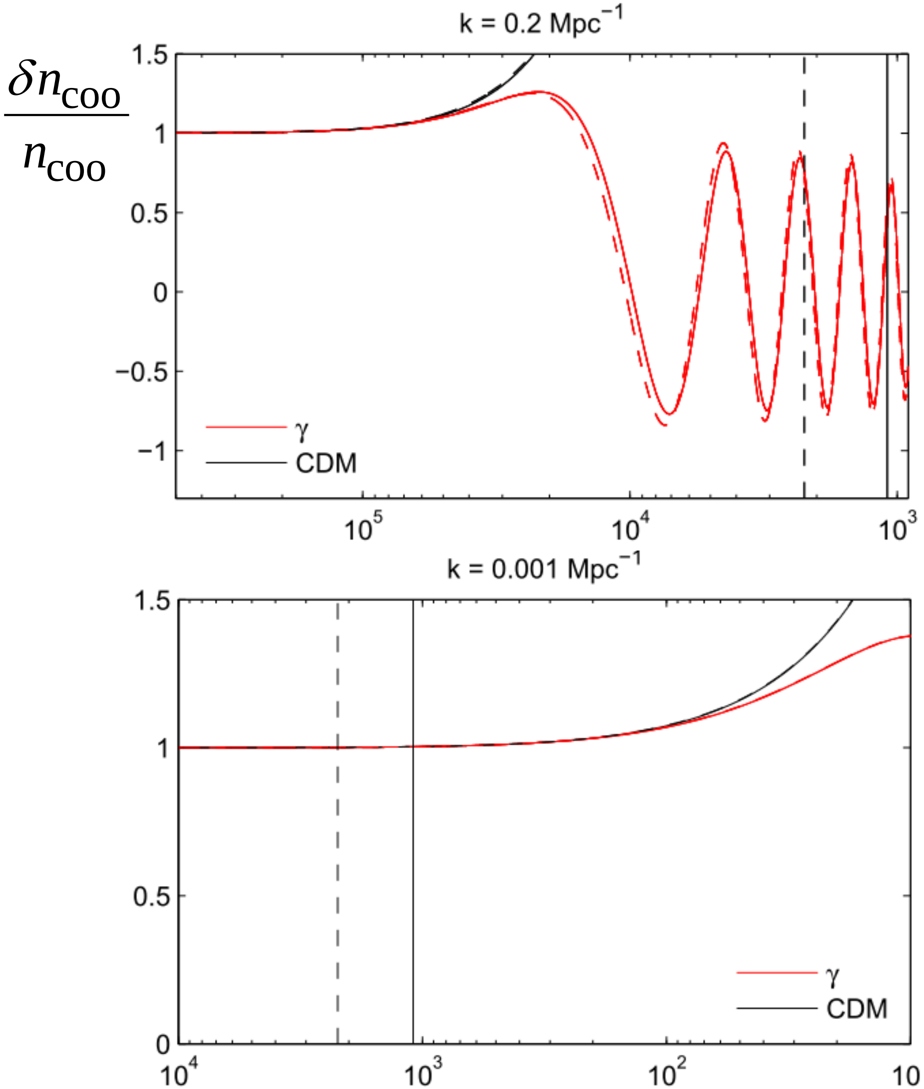
$$= -4(n_i\nabla_i\Phi - \dot{\Psi})$$

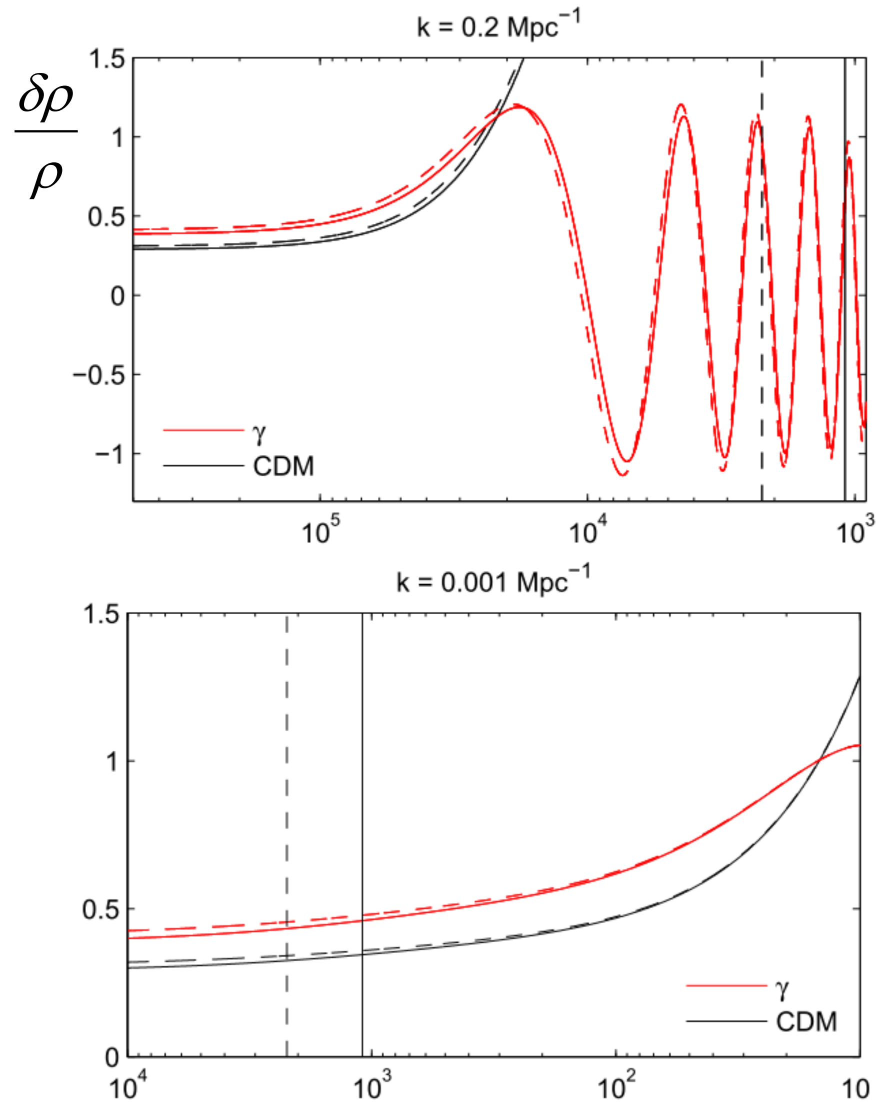
(Easy to include neutrino masses: SB, PRD 2006)

Simpler, more meaningful, more direct

- Ψ is a non-dynamical functional of δ_a and $\dot{\delta}_a$
- $\dot{\Psi}$ and $\ddot{\Psi}$ terms are dominant before and during horizon entry

This approach vs. Traditional





Simpler, more meaningful, connects microscopic properties with observables

Some effects on large and intermediate scales that were believed to be physical are artifacts of the traditional descriptions!

This approach vs. Traditional

- Analytical solutions (shown for the radiation era)

Photons:

$$\delta_\gamma = 3\zeta_{\rm in}\left(-\cos\varphi + \frac{2\sin\varphi}{\varphi}\right)$$

$$\delta_\gamma = 3\zeta_{\rm in}\left(-\cos\varphi + \frac{2\sin\varphi}{\varphi} + \frac{2\cos\varphi}{\varphi^2} - \frac{2\sin\varphi}{\varphi^3}\right)$$

Cold dark matter:

$$\delta_c = 6\zeta_{\rm in}\left(\ln\varphi + \gamma - \frac{1}{2} + \frac{\sin\varphi}{\varphi} - \mathrm{ci}\,\varphi\right)$$

$$\delta_c = 6\zeta_{\rm in}\left(\ln\varphi + \gamma - \frac{1}{2} + \frac{\sin\varphi}{\varphi} - \mathrm{ci}\,\varphi + \frac{\cos\varphi}{\varphi^2} - \frac{\sin\varphi}{\varphi^3}\right)$$

Simpler, more useful

$\varphi \equiv kS$, where $S(\tau) = \int_0^\tau c_s d\tau$ is sound horizon

This formalism produced for the first time analytically neutrino impact on the CMB:

SB and Seljak, PRD 2004

$$\delta_\gamma(k) \to A_\gamma \cos(kc_s\tau) - \pi\sqrt{3}\,(\Phi+\Psi)\big|_{x=c_s\tau} \sin(kc_s\tau)$$

It revealed that the phase of the CMB acoustic oscillations is shifted only by species whose perturbations propagate faster than sound.

Specifically, for neutrinos, $\delta l \approx -3.4\,\delta N_\nu$, for tracking quintessence $\delta l \approx -11\,\delta N_\phi$.

This approach lets us work in real space

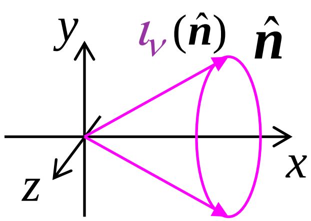
Photons:

$$\ddot{\delta}_\gamma - c_s^2 \nabla^2 \delta_\gamma = \nabla^2(\Phi + \Psi)$$

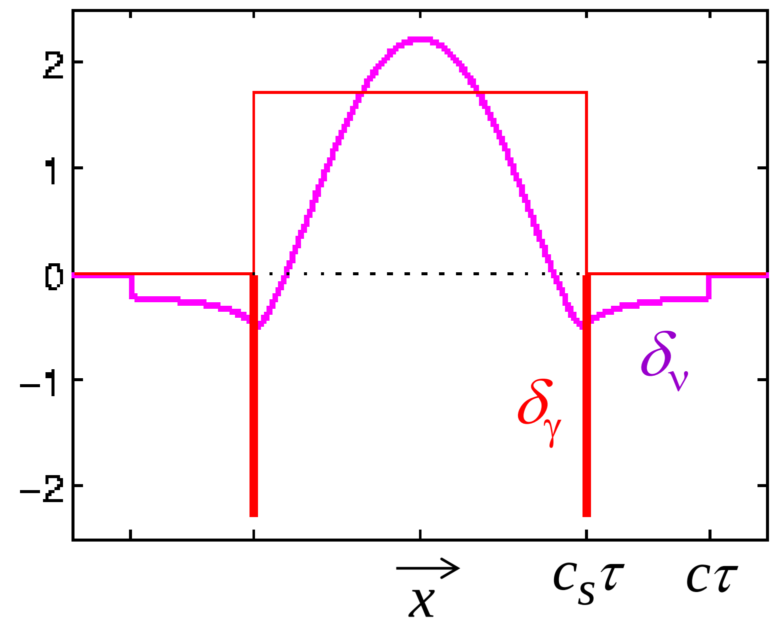
Neutrinos:

$$\dot{\imath}_\nu + n_i \nabla_i \imath_\nu = -4 n_i \nabla_i (\Phi + \Psi),$$

$$\delta_\nu = \langle \imath_\nu(\boldsymbol{n}) \rangle_{\boldsymbol{n}}$$



Analytically calculated Green's functions for the coupled CMB-neutrino evolution:



This calculation yielded the neutrino impact on the CMB (previous slide).

Conclusions

The suggested measures of cosmological perturbations obey much simpler, easier to integrate, and more physically meaningful dynamical equations than those of the traditional approaches.
The resulting equations manifest objective causal dependencies explicitly, whereas the traditional formalisms misguide our intuition about large-scale cosmological evolution.
The developed formalism lets us analyze realistic inhomogeneous evolution, intractable analytically with the earlier formalisms.
It has already led to the discovery of several previously unknown effects.*

Should this approach become mainstream?
Apparently. How soon will it?

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* In addition to the effects mentioned in the slides, see those described in SB arXiv:0707.0692 and in Baumann et al. arXiv:1803.10741