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MAPPING COSMOLOGICAL OBSERVABLES TO THE DARK KINETICS

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ABSTRACT

We study systematically which features in the cosmic microwave background (CMB) and large-scale structure (LSS) probe various inhomogeneous properties of the dark sectors (including neutrinos, dark matter, and dark energy). We stress, and quantify by simple formulas, that the primary CMB anisotropies are very susceptible to the gravitational potentials during horizon entry, less at recombination. The CMB thus allows us to scan $\Phi+\Psi$ and the underlying dark kinetics for all redshifts $z\sim 1-10^5$. LSS, on the other hand, responds strongest to Φ at low redshifts. Dark perturbations are often parameterized by the anisotropic stress and effective sound speed (stiffness). We find that the dark anisotropic stress and stiffness influence the visible species at the correspondingly early and late stages of horizon entry, and affect stronger respectively the CMB and LSS. The CMB yet remains essential to probing the stiff perturbations of light neutrinos and dark energy, detectable only during horizon entry. The clustering of dark species and large propagation speed of their inhomogeneities also map to distinctive features in the CMB and LSS. Any parameterization of the signatures of dark kinetics that assumes general relativity can effectively accommodate any modified gravity (MG) that retains the equivalence principle for the visible sectors. 4
This implies that formally the nonstandard structure growth or Φ/Ψ ratio, while indicative, are not definitive MG signatures. The definitive signatures of MG may include the strong dependence of the apparent dark dynamics on visible species, its superluminality, and the nonstandard phenomenology of gravitational waves. 8

Subject headings: cosmology: theory — cosmic microwave background — large-scale structure of universe — dark energy — modified gravity

1. INTRODUCTION

Galactic and cluster dynamics, cosmic structure, type Ia supernovae, the cosmic microwave background (CMB), and the primordial abundances of light elements provide solid evidence that dark sectors constitute a significant energy fraction of the universe at any accessible redshift $z\lesssim 10^{10}$. At all corresponding cosmological epochs the nature of abundant dark species, coupled to photons and baryons only by gravitation, is partly or entirely uncertain.

The mainstream analyses of cosmological data usually assume the minimal neutrino sector, non-interacting cold dark matter (CDM), and dark energy represented by a canonical scalar field (quintessence). These assumptions are reasonable for interpreting the available data, yet none of them can be taken for granted. For example, new light weakly interacting particles commonly appear in high-energy models. In some models, even the standard neutrinos recouple to each other or to additional light fields (Chacko et al. 2004, 2005; Beacom et al. 2004; Okui 2005; Grossman et al. 2005) at redshifts at which the decoupled component of radiation gravitationally affects the CMB and cosmic structure (Hu & Sugiyama 1996; Bashinsky & Seljak 2004; Hannestad 2005; Bell et al. 2006). Various alternatives to cold dark matter have been suggested as well. These include warm dark matter (Blumenthal et al. 1982; Olive & Turner 1982), self-interacting dark matter (Carlson et al. 1992; de Laix et al. 1995; Spergel & Steinhardt 2000), or modified gravity (Milgrom 1983; Bekenstein 2004; Skordis et al. 2006; Dodelson & Liguori 2006). The viability of such scenarios remains an intriguing question. Quintessence models are convenient for quantitatively constraining dark energy parameters by data. Yet quintessence is not readily motivated by particle physics, where it is difficult to naturally achieve the required shallowness of the field potential. On the other hand, many alternatives have been proposed whose inhomogeneous kinetics, and hence cosmological signatures, *cannot be mimicked by quintessence with any background* equation of state $w(z)$. (For a comprehensive review of dark energy models see, e.g., Copeland et al. 2006).

Fortunately, cosmological observations themselves can test these assumptions by revealing not only the dark species' mean density and pressure but also the kinetics of their inhomogeneities. The goal of this paper is to map various inhomogeneous kinetic properties of the dark sectors (or deviations from Einstein gravity) to the observable characteristics of CMB and cosmic structure. Dark species influence the visible matter by affecting both the background expansion and metric perturbations. Of the two mechanisms, the perturbations, albeit demanding better statistics for useful constraints, encode many more independent clues about the dark universe by offering *new information at every spatial scale k*. The following three examples show the importance of this information, absent in the background equation of state $w(z)$.

1.1. *Examples of the value of dark perturbations*

Nature of dark energy

The first example is the most challenging problem in today's cosmology—the nature of dark energy. The constraints on the dark energy background equation of state $w\equiv p_{\rm de}/\rho_{\rm de}$ are tightening around the value -1, consistent with a cosmological constant. Analyses that combine the current data from the CMB, large scale structure (LSS), Lyman-α forest, and supernovae, already constrain the deviation of w from -1 for flat models better than to 10% (Spergel et al. 2006; Seljak et al. 2006; Tegmark et al. 2006, and others.) Whether or not future observations continue to converge on $w=-1$, the dynamics of perturbations will be crucial in elucidating the nature of cosmic acceleration.

Even if $w(z)\equiv -1$ at low redshifts, this does not neces-

While the detection of such phenomena as non-standard growth of cosmic structure or anomalous lensing may indicate MG, we will see that without any restrictions on the dark dynamics, the identical effects could always be generated by a non-minimal dark sector that influences the visible matter according to the standard Einstein equations.[18] Further in this section we will argue that other features should yet allow to discriminate MG observationally.

We will assume that even if full Einstein gravity fails on cosmological scales, the Einstein principle of equivalence remains valid for the visible species. This assumption is common to many existing MG models. It is motivated by the relatively strong terrestrial and solar-system constraints on the equivalence principle.

Thus we suppose that the regular matter couples covariantly to a certain matter-frame "physical" metric $g_{\mu\nu}$. However, we now neither take for granted that all dark fields also couple covariantly to the same metric $g_{\mu\nu}$, nor assume that the dynamics of $g_{\mu\nu}$ itself is governed by the Einstein equations.

Under the weaker assumption of the equivalence principle for only the visible matter, all observable signatures of new
2,4 physics can still be quantified by *any of the three parameterization schemes* of Sec. 4. Indeed, the $g_{\mu\nu}$ background can still be described by a single number for its present spatial curvature and by its uniform redshift-dependent expansion rate $\mathcal{H}(z)$. The potentials Φ and Ψ, defined by eq. (3) to parameterize the inhomogeneities of the physical metric $g_{\mu\nu}$, will play their usual role in the evolution of light and baryons. Moreover, the (effective dark) energy and momentum densities assigned to the missing sources of curvature by the naive application of the Einstein equations will evolve in agreement with the usual local conser-
1 vation laws, which is easily seen as follows.

Let by definition

$$T^{\mu\nu}_{\text{eff dark}} \equiv \frac{1}{8\pi G}G^{\mu\nu} - \sum_{\text{known } a} T^{\mu\nu}_a, \qquad (46)$$

where the last sum is over the known regular particles. Their energy-momentum tensor [constructed unambiguously from
3.a the species' action S_a as $T^{\mu\nu}_a = (2/\sqrt{-g})\,\delta S_a/\delta g_{\mu\nu}$] is covariantly conserved by the assumed covariance for the regular species. The matter-frame Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, where $R_{\mu\nu}$ is the Ricci tensor of the physical metric, is also
3.b covariantly conserved by the Bianchi identities. Thus the entire expression (46) is covariantly conserved:

$$T^{\mu\nu}_{\text{eff dark};\nu} = 0, \qquad (47)$$

with all covariant derivatives being taken relative to the physical metric.

Since $T^{\mu\nu}_{\text{eff dark}}$ is covariantly conserved, the background and
2,4 perturbations of the missing energy and momentum evolve according to equations (16) – (19), derived from the identical conservation law. Thus if all our probes of the invisible degrees of freedom are based solely on their gravitational impact on light, baryons, and other regular particles (neutrinos, WIMP's when
5 probing dark energy, etc.) then phenomenologically *all observable signatures* of a considered MG model can be mimicked by an effective GR-coupled dark sector. Specifically, we can find the corresponding effective $w(z)$ to reproduce the missing energy background, and the effective anisotropic stress $\sigma(z,k)$ and stiffness $c^2_{\text{eff}}(z,k)$ to describe scalar perturbations. For example, for a popular DGP gravity model (Dvali et al. 2000) this was demonstrated explicitly by Kunz & Sapone (2007).

With this discouraging general conclusion, we may enquire whether cosmology can at all reveal definitive distinctions be-
tween MG and general relativity with a peculiar yet physically 6
permissible dark sector. To establish such distinctions, let us summarize the conceptual differences of MG from general relativity:

A. Some dark degrees of freedom may not couple covariantly to the matter metric $g_{\mu\nu}$. 7

B. The gravitational action may not be given entirely by the Hilbert-Einstein term $S_{\text{grav}} = (16\pi G)^{-1}\int d^4x\sqrt{-g}R$.

The distinctive observable consequences of these special properties of MG may include:

1. The effective dark dynamics, which is observationally inferred by assuming the Einstein equations, violates the equivalence principle (EP). The EP violation can be 8
seen, e.g., as
    i. The dependence of the inferred local dark dynamics on the distribution of visible matter, when it cannot be explained by non-gravitational dark–visible coupling allowed by particle experiments.
    ii. Superluminality of the inferred dark dynamics.
2. The dynamics of gravitational waves (tensor modes) deviates from the predictions of the Einstein equations, assuming that both the visible and inferred dark species contribute to the energy-momentum tensor in the simplest way.

Most of these signatures have already been utilized for falsifying MG models with existing or suggested observations, (Clowe et al. 2006; Bradac et al. 2006) and (Kahya & Woodard 2007); we comment additionally on them next.

6.1. *EP violation for the inferred dark dynamics*

The violation of the first condition can be illustrated by an extreme toy theory in which the regular matter ("baryons," for short) constitute all independent degrees of freedom. Let the metric in this theory be specified by baryon distribution via some deterministic relation (e.g., as $g^{\mu\nu} = \Lambda^{-1}T^{\mu\nu}_{\text{baryon}}$, following from an action $S = S_{\text{baryon}} - \int d^4x\sqrt{-g}\,\Lambda$ with Λ being a constant). Even in such a contrived theory, by the above arguments, the effective missing energy and momentum densities (46) would appear to evolve and gravitate in agreement with energy-momentum conservation and the Einstein equations. In this example, however, the effective dark density and stress are uniquely determined by the distribution of the visible matter. This does not occur for truly independent dark degrees of freedom.

In more realistic MG theories we should not expect a deterministic relation between the visible and effective dark distributions. Still, if the dark and visible sectors interact other than by coupling covariantly to the common metric then the inferred
laws of the effective local dark dynamics would depend on the 8
visible environment.

Detection of such dependencies would be particularly feasible for the dark matter, for which there are plentiful observable regions with varying environment: varying in both visible matter density and in its ratio to dark density. In addition

[18] We stress that these features remain useful hints of MG. Since they are even more ubiquitous than the scenarios of modified gravity and may reveal other new physics, they are well worth searching for.

TABLE 1

Property	Quantified by	Important for	Effect on the CMB	Effect on Matter
Anisotropic Stress	σ, eq. (13) [$\Phi-\Psi$, eq. (14)]	Early stage of horizon entry	Amplitude (Suppressed by σ from streaming)	Minor on power (Enhanced by σ from streaming)
Stiffness	$c^2_{\rm eff}$, eq. (15)	Late stage of horizon entry	Amplitude (Enhanced by tracking quintessence)	Medium on power (Suppressed by tracking quintessence)
Velocity of a perturbation front	c_p, Sec. 5.3	Features local in real space	Phase of the acoustic peaks	Phase of baryonic oscillations
Self-clustering	Φ, $\Phi+\Psi$ eq. (3)	Horizon entry (CMB) and subhorizon evolution (LSS)	Significant suppression of the amplitude	Primary driving of the structure growth

NOTE.—Summary of the discussed properties of the dark sectors, the epochs of their observational relevance, and their effects on the CMB power spectra and on large-scale structure.

inhomogeneities of radiation before equality.

The suppression of C_ℓ for $\ell \lesssim 100$ severely restricts the alternatives to CDM and the models of dark energy which reduce metric perturbations at any redshift in the matter era. Examples of such mechanisms are contribution of quintessence to the density in the matter era, interaction or unification of dark matter and dark energy (e.g., Wetterich 1988; Perrotta & Baccigalupi 2002; G. R. Farrar, P. J. Peebles 2004; Catena et al. 2004; Scherrer 2004), or MOND-inspired alternatives to dark-matter (Milgrom 1983; Bekenstein 2004).

7.4. *Modified gravity*

Many authors have suggested that modification of general relativity on cosmological scales is the cause of the cosmic acceleration (for recent reviews see Copeland et al. 2006; Nojiri & Odintsov 2007) or even of the apparent manifestations of dark matter (Milgrom 1983; Bekenstein 2004). In Sec. 6 we consider the phenomenology of typical models of modified gravity (MG) that retain the equivalence principle for the visible sectors. 2,4 We show that in these models all gravitational impact of the hidden physics can be described within the same parameterization schemes of Sec. 4, developed to quantify the observable properties of dark sectors that are coupled by general relativity (GR). Indeed, these schemes were restricted only by the covariance of the visible dynamics, the assumption of the Einstein equations, and the local conservation of the dark energy and momentum. 3 However, for any covariant visible dynamics, the formal dark energy-momentum tensor (46) that is missing in the Einstein equations is covariantly conserved automatically (Sec. 6). 5 Thus all observable signatures of MG can be mimicked by effective dark energy and momentum that influence the visible species according to the Einstein equations and during evolution are conserved locally.

Particularly, the nonstandard structure growth or Φ/Ψ ratio that are predicted by *any* MG model of the considered broad class can in principle be reproduced without violation of the Einstein equations by sufficiently peculiar dark dynamics. Nevertheless, first, such signatures would still signal some non-minimal physics and therefore should be considered for experimental constraints whenever possible. Second, GR remains falsifiable by other effects; 6 in particular, by the violation of the equivalence principle by apparent dark dynamics. This may be manifested in the strong dependence of the dark dynamics on the visible matter (Sec. 6.1), 8 and in the signatures of effective or real superluminal dark flows (e.g., Sec. 6.2). GR can also be falsified by nonstandard phenomenology of gravitational waves (e.g. Kahya & Woodard 2007, and Secs. 6.2 and 6.3).

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APPENDIX

CMB SENSITIVITY TO $\delta\Phi$ ON SMALL SCALES

We quantify the CMB sensitivity to the metric inhomogeneities on subhorizon scales and at late times using the Limber approximation (Limber 1954), applied to the CMB by Kaiser (1984, 1992) and Hu & White (1996). The Limber approximation for the CMB power spectrum C_l assumes that the change of the source S in the line-of-sight integral (31) is negligible over the wavelength of a typical contributing mode $k^{-1} \sim r/\ell$. If $\delta\tau$ is a temporal scale over which the source changes by an order of unity then the above condition is equivalent to $k\,\delta\tau \gg 1$. In the Limber limit, C_l is primarily contributed by the modes with $|\mathbf{k}\cdot\mathbf{n}|\,\delta\tau \lesssim 1$, while positive and negative contributions to $\Delta T/T$ from the peaks and troughs of the other modes cancel (Hu & White 1996).

The ISW contribution to the anisotropy source (32) is

$$S_{\rm ISW} = g\,(\dot\Phi+\dot\Psi). \qquad \text{(A1)}$$